



Measures of Central Tendency -
Arithmetic mean, Geometric
mean, Harmonic mean

Arithmetic Mean (AM)

The Arithmetic Mean is the “regular average” — it assumes all values contribute equally and additively. It answers:

☞ “If everything were evenly distributed, what would each value be?”

Formula

For individual data:

AM =

- N
- ΣX

For frequency data:

- AM =
- Σf
- ΣfX

Business Interpretation

- Used when values are independent and additive

Ideal for:

- Average sales
- Average income
- Average cost

Example

- If daily sales = 100, 150, 200
- $AM =$
- 3
- $100+150+200=150$

Merits & Demerits

Merits

- Simple and widely understood
- Uses all observations
- Algebraically tractable (important for further analysis)

Demerits

- Highly affected by extreme values (outliers)
- Not suitable for skewed distributions

Geometric Mean (GM)

Concept (Intuition)

- GM is the average of multiplicative or proportional changes.

↳ It answers: “What constant growth rate would produce the same overall effect?”

- Formula
- $GM = \sqrt[N]{\prod X}$
- N
- 1
- For frequency data:
- $GM = \text{Antilog}$
- Σf
- $\Sigma f \log X / N$

- Business Interpretation
 - Used for:
 - Growth rates (population, sales, investment returns)
 - Index numbers
 - Compound interest
 - Example
 - Growth rates: 10%, 20%, 30%
 - $GM=(1.1 \times 1.2 \times 1.3)$
 - $1/3$
- $\approx 1.197 \Rightarrow 19.7\%$

Merits & Demerits

Merits

- Best for percentage changes
- Less affected by extreme values than AM
- Reflects compounding effect

Demerits

- Cannot be used if any value is zero or negative
- More complex to compute

Harmonic Mean (HM)

Concept (Intuition)

- HM is used when values are related to rates or ratios, especially when the variable is in the denominator.

↳ It answers: "What is the average rate when the same quantity is involved?"

• Formula

- $HM = \frac{1}{\frac{1}{N} \sum \frac{1}{X}}$

For frequency data:

- $HM =$

- \sum

- X

- f

- $\sum f$

- Business Interpretation
- Used for:
- Average speed
- Price per unit
- Efficiency ratios
- Example
- Speed: 60 km/h and 40 km/h for equal distances
- $HM =$
- 60
- 1+
- 40
- 1
- 2
- =48 km/h

Merits & Demerits

Merits

- Best for averaging rates
- Gives more weight to smaller values

Demerits

- Sensitive to very small values
- Not suitable for general data

Relationship Between AM, GM, and HM

- $AM \geq GM \geq HM$
- Equality holds when all values are equal.
- Interpretation
- AM = optimistic average
- GM = realistic average (growth-based)
- HM = conservative average (constraint-based)

Practical Business Applications

1. Finance

- AM: Average return (simple)
- GM: Compound annual growth rate (CAGR)

2. Operations

- HM: Average production rate

3. Economics

- GM: Index numbers
- AM: Income distribution

Comparative Business Perspective

<i>Measure</i>	<i>Nature</i>	<i>Best Use Case</i>	<i>Sensitivity</i>
<i>AM</i>	<i>Additive</i>	<i>Salaries, sales</i>	<i>High</i>
<i>GM</i>	<i>Multiplicative</i>	<i>Growth rates</i>	<i>Moderate</i>
<i>HM</i>	<i>Reciprocal</i>	<i>Speeds, ratios</i>	<i>Sensitive to small values</i>

Final Conceptual Insight

Think of the three means as different “lenses”:

- AM → Equal sharing (fair distribution mindset)
- GM → Compounding (growth mindset)
- HM → Efficiency (constraint/bottleneck mindset)

No single mean is “best” — the choice depends on data nature and business objective.